using a heater-burner to simulate the engine exhaust, seemed to indicate that the two-dimensional CD nozzle with exit aspect ratios less than 10 would not produce less IR signature than the equivalent axisymmetric nozzle (see Fig. 67 of Ref. 5). The effect of the engine swirl is expected to be more important when external streams are present, since in such cases external mixing due to shear is otherwise reduced because of the lowered velocity differences between the plume and external stream.

In conclusion, a simple modeling technique has been developed[1,2] to provide economical yet adequate prediction of two-dimensional-nozzle plume properties for IR signature analysis. Successful models were developed for ADEN, two-dimensional CD, and two-dimensional plug nozzles with and without bypass flow and/or engine swirl. A 10-deg engine swirl was found to reduce the predicted plume temperature dramatically. Therefore the inherent engine swirl, though undesirable from the performance point of view, appears to be highly effective in reducing the IR signature of a two-dimensional-nozzle plume.

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Take-Off Ground Roll of Propeller Driven Aircraft

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Nomenclature

C_D = drag coefficient
C_L = lift coefficient
D = reference length
g = acceleration due to gravity
k = induced drag factor, $C_D = C_{D_0} + kC_L^2$
m = airplane mass
P = effective thrust power
S = wing area
s_D = aerodynamic penetration, $s_D = 2m/\rho SC_D$
s_L = aerodynamic radius, $s_L = 2m/\rho SC_L$
T = thrust
V = speed
V_r = reference speed
V_{TO} = take-off speed
$\bar{V}$ = speed ratio, $\bar{V} = V/V_r$
W = airplane weight
x = ground roll distance
$\bar{x}$ = distance ratio, $\bar{x} = x/D$
μ = coefficient of rolling resistance
ρ = air density
ϕ = airplane parameter

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Introduction

In aircraft performance analysis take-off ground roll calculations are usually performed numerically. In preliminary design, however, it is convenient to have available simple analytical techniques for estimating the take-off ground roll distance.

When the engine performance is expressed in terms of thrust the ground roll solution is well known.[1,2] For engine thrust independent of airspeed the ground roll distance is

$$x = (1/2B)\ln[A/(A - BV_{T_0}^2)] \tag{1}$$

where

$$A = g(T/W - \mu) \tag{2}$$

$$B = (C_D - \mu C_L) g\rho S/2W \tag{3}$$

The ground roll is minimized when $C_D - \mu C_L$ is minimum or when

$$C_L = \mu/2k \tag{4}$$

The solution given by Eqs. (1-3) does not apply to propeller driven aircraft since, in this case, the thrust-power, rather than the thrust, is constant. Thus the actual thrust will be inversely proportional to the speed. The solution to the acceleration problem for constant power with aerodynamic drag was first developed for automobile performance. Pershing[3] considered only aerodynamic drag while Hawks and Sayre[4] included both drag and lift in their analysis. In this Note the method of Ref. 4 will be applied to the take-off ground roll problem for propeller driven aircraft.

Theory

If we assume that the effective thrust-power of the engine-propeller combination is independent of airspeed the horizontal equation of motion for the airplane during ground roll (Fig. 1) is

$$mV\frac{dV}{dx} = \frac{P}{V} - \mu mg - \frac{1}{2}\rho S(C_D - \mu C_L)V^2 \tag{5}$$

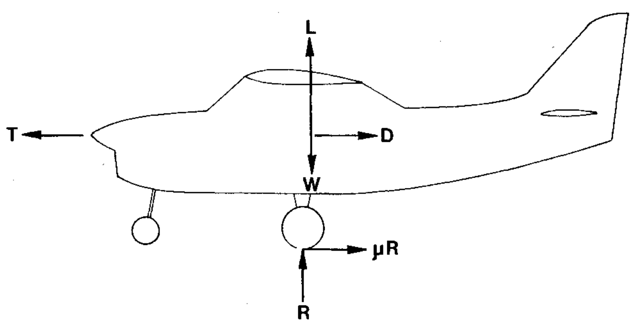

Fig. 1 Forces on aircraft in ground roll.

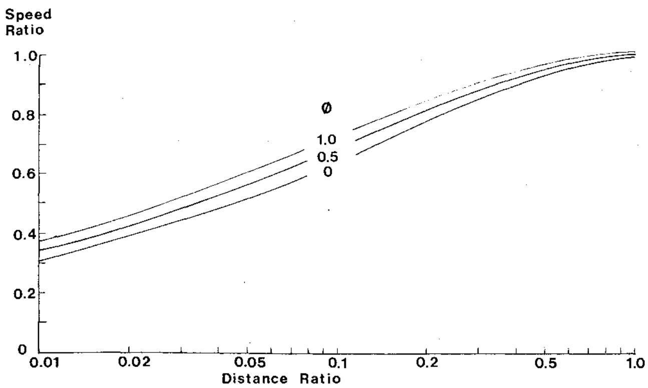

Fig. 2 Distance solution for power limited acceleration.

Table 1 Properties of sample aircraft

W	56,000 lb
S	1000 ft^2
C_{D_0}	0.024
k	0.04
μ	0.025
P	3600 hp
T	13,000 lb (static)
V_{T_0}	168 ft/s

Defining the reference distance D to be

$$D = s_L s_D / (s_L - \mu s_D) \tag{6}$$

where s_L and s_D are the aerodynamic radius and aerodynamic penetration,[5] respectively, reduces Eq. (5) to

$$V\frac{\mathrm{d}V}{\mathrm{d}x} + \frac{V^2}{D} - \frac{P}{mV} + \mu g = 0 \tag{7}$$

Even though it is physically impossible to attain we define the reference speed V_r to be the equilibrium solution of Eq. (7). That is, V_r is given by the single positive root of

$$V_r^3 + \mu g D V_r - PD/m = 0 \tag{8}$$

Normalizing the speed to V_r and the distance to D results in the dimensionless equation

$$\bar{V}^2\frac{\mathrm{d}\bar{V}}{\mathrm{d}\bar{x}} + \bar{V}^3 + \phi\bar{V} = 1 + \phi \tag{9}$$

where

$$\phi = \mu g D / V_r^2 \tag{10}$$

Equation (9) can be integrated to obtain

$$\bar{x}(\phi+3) = \ln\left(\frac{1-\bar{V}_0}{1-\bar{V}}\right) - \tfrac{1}{2}(\phi+2)\ln\left(\frac{\bar{V}^2+\bar{V}+\phi+1}{\bar{V}_0^2+\bar{V}_0+\phi+1}\right) + \frac{\phi}{2(\phi+\frac{3}{4})^{1/2}}\tan^{-1}\left(\frac{(\bar{V}_0-\bar{V})(\phi+\frac{3}{4})^{1/2}}{\bar{V}\bar{V}_0+\frac{1}{2}(\bar{V}+\bar{V}_0)+\phi+1}\right) \tag{11}$$

with $\bar{V}_0$ being the initial value of the speed ratio. Figure 2 is a plot of Eq. (11) for $\bar{V}_0$ equal to zero. This solution can be applied to any aircraft by finding the values of V_r and D for the vehicle.

From Fig. 2 it is apparent that the ground roll distance for any take-off speed is minimized by maximizing ϕ. This is the same as maximizing D or minimizing $C_D - \mu C_L$. Thus the lift coefficient for minimum ground roll has the same value for constant power that it has for constant thrust.

Application

To illustrate the use of Eq. (11) consider the airplane described in Table 1. For minimum ground roll C_L is given by Eq. (4) as 0.3125 which makes $C_D = 0.0279$. The aerodynamic penetration and radius at sea level are then $s_D = 52{,}475$ ft and $s_L = 4685$ ft. This makes the reference distance $D = 72{,}884$ ft.

The reference speed will be given by the equation

$$V_r^3 + 58617V_r - 82.9\times10^6 = 0$$

which has the positive root $V_r = 391.4$ ft/s. From Eq. (10) we find $\phi = 0.383$. At take-off speed $\bar{V} = 0.429$. Using Eq. (11) (or from Fig. 2) $\bar{x} = 0.02165$. Therefore the minimum take-off ground roll is 1578 ft.

If Eq. (1) were used with the average thrust value of 12,400 lb the minimum distance would be 2305 ft. Of course, neither answer will accurately describe the actual performance of the aircraft since a propeller driven aircraft does not operate with constant power or with constant thrust during the take-off ground roll.

The major difficulty with the constant power calculation is that the static thrust would have to be infinite to maintain true constant power. This problem can be avoided by treating the thrust as constant at the static thrust value up to the speed where the static thrust would produce the rated thrust-power and the power as constant above that speed. In our example we would then use Eq. (1) for ground roll from 0 to 152.3 ft/s and Eq. (11) for speeds from 152.3 ft/s to V_{T_0}. This method gives a minimum ground roll distance of 2207 ft.

It is not the purpose of this Note to advocate any one method of solving the ground roll problem since all analytic methods involve some sort of approximation. The intent rather is to point out that simple analytic solutions do exist for power-limited acceleration with both lift and drag acting. This does provide the analyst with an additional technique for preliminary estimation of the take-off ground roll distance of propeller driven aircraft.

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